The Stringy Representation of the $D \geq 3$ Yang-Mills Theory.

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Abstract

I put forward the stringy representation of the $1/N$ strong coupling (SC) expansion for the regularized Wilson's loop-averages in the *continuous* $D \geq 3$ Yang-Mills theory (YM_D) with a sufficiently large *bare* coupling constant $\lambda > \lambda_{cr}$ and a fixed ultraviolet cut off Λ. The proposed representation is proved to provide with the *confining* solution of the Dyson-Schwinger chain of the judiciously *regularized* $U(N)$ Loop equations. Building on the results obtained, we suggest the stringy pattern of the low-energy theory associated to the $D = 4$ $U(\infty) \cong SU(\infty)$ gauge theory in the standard $\lambda \to 0$ phase with the asymptotic freedom in the UV domain. A nontrivial test, to clarify whether the AdS/CFT correspondence conjecture may be indeed applicable to the large N pure YM_4 theory in the $\lambda \to \infty$ limit, is also discussed.

Keywords: Yang-Mills, Loop equation, Duality, String, Strong-coupling expansion

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1 Introduction.

There is some evidence [1, 2] that the $D \geq 3$ pure Yang-Mills systems (YM_D), including the conventional one with the action

$$S = \frac{1}{4g^2} \int d^D x \; tr\, (F_{\mu\nu}(x) F_{\mu\nu}(x)) \ , \tag{1.1}$$

might be reformulated as a sort of string theory. Recently, synthesizing the nonabelian duality transformation [4] with the Gross-Taylor stringy reformulation [3] of the $D = 2$ YM_2 theory, a concrete proposal [5] has been made for the stringy representation of the $1/N$ strong coupling (SC) expansion in a generic continuous $D \geq 3$ YM_D system with a fixed ultraviolet (UV) cut off Λ. The construction is built on the key-observation revealing the exact WC/SC *correspondence* which takes place (prior to the UV regularization) between the two alternative $1/N$ series. Certain conglomerates of the Feynman diagrams, comprising the $1/N$ weak-coupling (WC) series in a given continuous gauge system, via a formal resummation can be traded for the judiciously associated variety of the appropriately weighted (piecewise) smooth worldsheets of the color-electric flux, facilitating the $1/N$ SC expansion in the same YM_D system.

Given the proper regularization (that introduces a *nonzero* width of the YM vortex), a particular variant of the latter SC expansion is supposed to faithfully represent, at least for sufficiently large N, the regularized $U(N)$ gauge theory (1.1) provided that the dimensionless and N*-independent bare* coupling constant

$$\lambda = (g^2 N) \Lambda^{D-4} > \lambda_{cr}(D) \tag{1.2}$$

is *larger* than certain critical value $\lambda_{cr}(D)$ presumably associated to the large N phase transition. For the final justification of the asserted $YM_D/String$ duality, in the present paper I follow the somewhat complementary root. Given the $U(N)$ theory (1.1), our aim is to solve the Dyson-Schwinger chain of the loop equations [7, 8] which, in the $N \to \infty$ limit, reduces to the single Loop equation

$$\hat{\mathcal{L}}_\nu(\mathbf{x}(s)) < W_C >_\infty = \tilde{g}^2 \oint_C dy_\nu(s') \; \delta_D(\mathbf{y}(s') - \mathbf{x}(s)) < W_{C_{xy}} >_\infty < W_{C_{yx}} >_\infty, \tag{1.3}$$

where $\tilde{g}^2 = g^2 N = \lambda \Lambda^{4-D}$, $\hat{\mathcal{L}}_\nu(\mathbf{x})$ is the Loop operator specified by eq. (2.5) below, while $< W_C >_\infty$ denotes the $N \to \infty$ limit of the correlator of the Wilson loop[1]

$$W_C = \frac{1}{N} Tr \Big[\mathcal{P} \; exp\Big(i \oint_C dx_\mu A_\mu(x) \Big) \Big] \quad , \quad A_\mu(x) \equiv A^a_\mu(x) T^a_{ij} \ . \tag{1.4}$$

It can be then demonstrated, see [6], that the derived $D \geq 3$ *confining* solution (of the loop equations) matches, to all orders of the $1/N$ expansion, the appropriately regularized implementation of the Gauge String proposed in [5].

The subtlety is that, in the SC phase (1.2), the transverse profile of the *microscopic* flux-tubes does depend on a particular choice of the prescription to implement the UV regularization. Therefore, to attack the loop equations in the economic way, it is vital to reduce the regularization-dependence of the YM_D loop-average $< W_C >$ as much as possible. Upon a reflection, there are

[1] Owing to the constraint imposed by the D-dimensional $\delta_D(..)$-function, the r.h. side of eq. (1.3) vanishes unless C has a selfintersection at a point $\mathbf{x}(s) = \mathbf{y}(s')$ so that $C \equiv C_{xx} = C_{xy} C_{yx}$ is decomposable into the two subloops C_{xy} and C_{yx}.

two limiting regimes (formalized by eqs. (1.5) and (1.6) below) where the dependence of $< W_C >$ on the choice of the regularization indeed can be reduced to the dependence of a few relevant coupling constants, entering the corresponding formulation of the Gauge String representation, on the bare coupling (1.2). In both cases, we are dealing with the dominance of the *infrared* phenomena: the contours C, being constrained to possess the radius of curvature $\mathcal{R}(s) >> \Lambda^{-1}$ (for $\forall s$), should be associated to sufficiently large values of the minimal area $A_{min}(C)$ of the saddle-point worldsheet $\tilde{M}_{min}(C)$ spanned by C. Given the latter conditions, the required reduction is shown to be maintained when the characteristic amplitude $\sqrt{< \mathbf{h^2} >}$ of the worldsheet's fluctuations is *either* much larger

$$\frac{< \mathbf{h^2} >}{< \mathbf{r^2} >} \sim \frac{D-2}{\lambda} \cdot ln[A_{min}(C)\Lambda^2] \longrightarrow \infty \quad , \quad \mathcal{R}(s)\Lambda \longrightarrow \infty \, , \tag{1.5}$$

or much smaller

$$\frac{< \mathbf{h^2} >}{< \mathbf{r^2} >} \sim \frac{ln[A_{min}(C)\Lambda^2]}{\lambda} \longrightarrow 0 \quad , \quad [A_{min}(C)\Lambda^2] \longrightarrow \infty \quad , \quad \mathcal{R}(s)\Lambda \longrightarrow \infty \, , \tag{1.6}$$

than the flux-tube's width $\sqrt{< \mathbf{r^2} >} \sim \Lambda^{-1}$.

In both of the above regimes, the corresponding solution of the Dyson-Schwinger chain allows to reconstruct the associated regularization of Gauge String representation [5] (of the Wilson's loop-averages). The prescription is that the 'bare' worldsheet's weight (assigned to the infinitely thin flux-tubes) of [5] is substituted by its smeared counterpart, visualized through the *fat* YM vortices, so that the nonuniversality of the smearing is essentially unobservable under the considered conditions (1.5) or (1.6). The discussed solution is expected to describe *stable* stringy excitations only in the SC phase (1.2). At least for sufficiently large N, this is predetermined by the fact that, within the $1/N$ SC expansion, the physical string tension σ_{ph} mandatory exhibits the Λ^2-scaling (2.11). Nevertheless, combining the present results with the outcome of [5], we suggest the stringy pattern of the low-energy theory associated to the $D = 4$ $U(\infty) \cong SU(\infty)$ gauge theory (1.1) in the conventional WC phase $\lambda \rightarrow 0$.

2 The regime (1.5) of the large fluctuations.

The complexity of eq. (1.3) is foreshadowed by its *nonlinearity* that takes place when the loop C nontrivially selfintersects. Complementary, one may expect that, in eq. (1.3), *not* any 'natural' (from the $\lambda \rightarrow 0$ phase viewpoint) regularization of the $\delta_D(\mathbf{x}-\mathbf{y})$-function can be translated into a *tractable* regularization of the presumable stringy solution of the Loop equation in the SC phase. On the contrary, in the sector Υ_0 (of the full loop space Υ) comprised of the contours C *without* nontrivial selfintersections, eq. (1.3) considerably simplifies so that a simple gauge-invariant regularization results in the transparent pattern of the regularized stringy solution. To take advantage of the latter simplification, the idea is to handle the Loop equation (1.3) in the two steps. First, one is to find a subclass of the regularized solutions of the considered Υ_0-reduction of the Loop equation. Then, we find out under what circumstances thus obtained solutions correctly reproduce the loop-averages for nontrivially *selfintersecting* contours.

2.1 The solutions of the Loop Equation on Υ_0.

For any nonintersecting loop $C \in \Upsilon_0$, the r.h. side of eq. (1.3) receives a nonzero contribution only from the *trivial* selfintersection point $\mathbf{x}(s) = \mathbf{y}(s')$ with $s' = s$ so that one can put $<$

$W_{C_{xy}} >=< W_{C_{xx}} >$ while $< W_{C_{yx}} >= 1$. Actually, the same simplification takes place for the finite N extension of eq. (1.3) so that the latter finite N equation is reduced on Υ_0 to the *linear* one

$$\hat{\mathcal{L}}_\nu(\mathbf{x}(s)) < W_C >= \tilde{g}^2 < W_C > \oint\limits_C dy_\nu(s') \ \delta_D(\mathbf{y}(s') - \mathbf{x}(s)) \quad , \quad C \in \Upsilon_0 \ , \tag{2.1}$$

where $\tilde{g}^2 = g^2 N \sim N^0$, and one can implement (consistently with the manifest gauge invariance) the smearing prescription

$$\delta_D(\mathbf{x} - \mathbf{y}) \ \longrightarrow \ \Lambda^D \mathcal{G}(\Lambda^2 (\mathbf{x} - \mathbf{y})^2) \quad ; \quad \int d^D z \ \mathcal{G}(\mathbf{z}^2) = 1 \ , \tag{2.2}$$

where $\mathcal{G}(\mathbf{z}^2)$ is a sufficiently smooth function (so that all its moments are well-defined) which satisifies the natural normalization-condition.

Next, a priori, one can search for the solution of the reduced eq. (2.1) in the form of the regularized sum

$$< W_C >= \sum_{\tilde{M}}^{(r)} \ w_2[\tilde{M}(C)] \tag{2.3}$$

over the worldsheets $\tilde{M}(C)$ weighted by a factor $w_2[\tilde{M}(C)]$. Akin to the conventional Nambu-Goto theory, the surfaces $\tilde{M}(C)$ are supposed to result from the smooth immersions into the Euclidean base-space $\mathbf{R^D}$, and the sum's superscript (r) recalls about the UV cut off Λ for the transverse fluctuations of the string. Then, plunging the stringy Ansatz (2.3) into eq. (2.1), the latter equation can be transformed into the one (to be regularized according to eq. (2.2))

$$\hat{\mathcal{L}}_\nu(\mathbf{x}(s)) \ ln \left(w_2[\tilde{M}(C)] \right) = \tilde{g}^2 \oint\limits_C dy_\nu(s') \ \delta_D(\mathbf{y}(s') - \mathbf{x}(s)) \tag{2.4}$$

which operates directly with the worldsheet's weight $w_2[\tilde{M}(C)]$. In going over from eq. (2.1) to eq. (2.4), we have taken into account that the Loop operator[2]

$$\hat{\mathcal{L}}_\nu(\mathbf{x}(s)) = \partial_\mu^{\mathbf{x}(s)} \ \frac{\delta}{\delta \sigma_{\mu\nu}(\mathbf{x}(s))} \tag{2.5}$$

complies with the *Leibnitz* rule. Due to the first-order nature of $\hat{\mathcal{L}}_\nu$, the general solution $w_2[\tilde{M}(C)]$ of eq. (2.4) assumes the form

$$w_2[\tilde{M}(C)] = \tilde{w}_2[\tilde{M}(C)] \ w_2^{(0)}[\tilde{M}(C)] \quad ; \quad \hat{\mathcal{L}}_\nu(\mathbf{x}(s)) \ ln(w_2^{(0)}[\tilde{M}(C)]) = 0, \tag{2.6}$$

where $\tilde{w}_2[..]$ is any particular solution of eq. (2.4), while $w_2^{(0)}[\tilde{M}(C)]$ is formally allowed to be an arbitrary N*-independent zero mode* (of the Loop operator) fulfilling some natural cluster decomposition requirement.

As it is demonstrated in [6] (where the general solution of the zero-mode equation (2.6) is found), the zero-mode factor does not alter the picture arising when one puts $w_2^{(0)}[\tilde{M}(C)] = 1$ retaining only the appropriate particular solution $\tilde{w}_2[..]$. To derive the latter, it is helpful to utilize the abelian Stokes theorem and rewrite the r.h. side of eq. (2.4) as the surface integral over $\tilde{M}(C)$. Then, after some simple manipulations, one can reduce (see [6] for the details) eq. (2.4) to the transparent equation

$$\frac{2 \ \delta^2 \ ln \left(\tilde{w}_2[\tilde{M}(C)] \right)}{\delta \sigma_{\mu\nu}(\mathbf{x}(s)) \ \delta \sigma_{\rho\chi}(\mathbf{y}(s'))} = -(\delta_{\mu\rho}\delta_{\nu\chi} - \delta_{\mu\chi}\delta_{\nu\rho}) \ \lambda \Lambda^4 \ \mathcal{G}(\Lambda^2 (\mathbf{x} - \mathbf{y})^2) \ , \tag{2.7}$$

[2] In eq. (2.4), $\delta/\delta\sigma_{\mu\nu}(\mathbf{x}(s))$ and $\partial_\mu^{\mathbf{x}(s)}$ denote respectively the Mandelstam area-derivative and the path-derivative [7].

Finally, the structure of eq. (2.7) is suggestive that the Mandelstam area-derivatives might be traded, $\delta/\delta\sigma_{\mu\nu}(\mathbf{x}(s)) \to \delta_f/\delta p_{\mu\nu}(\mathbf{x}(\gamma))|_{\mathbf{x}(\gamma)=\mathbf{x}(s)\in C}$, for the ordinary functional area-derivatives (preliminary restricted to the boundary C) with respect to the standard infinitesimal area-element $d\sigma_{\mu\nu}(\mathbf{x}(\gamma)) = p_{\mu\nu}(\gamma)\ d^2\gamma,\ p_{\mu\nu}(\gamma) = \varepsilon^{ab}\ \partial_a x_\mu(\gamma)\partial_b x_\nu(\gamma)$, where the coordinates $x_\mu(\gamma) \equiv x_\mu(\gamma_1, \gamma_2)$ define the position of a given worldsheet $\tilde{M}$ in the base-space $\mathbf{R^D}$. The considered substitution is supported by the pattern of the resulting solution[3]

$$\tilde{w}_2[\tilde{M}_\chi] = N^\chi\ exp\left(-\frac{\lambda\Lambda^2}{4} \int\limits_{\tilde{M}_\chi}\int\limits_{\tilde{M}_\chi} d\sigma_{\mu\nu}(\mathbf{x})d\sigma_{\mu\nu}(\mathbf{y})\ \Lambda^2\mathcal{G}(\Lambda^2(\mathbf{x}-\mathbf{y})^2) \right)\ , \tag{2.8}$$

where, advancing ahead, we have included the 't Hooft topological factor (with χ being the total Euler character of $\tilde{M}_\chi$) which justification requires to deal with the full chain of the loop equations.

Actually, the quasi-local pattern (2.8) can be linked, despite its somewhat bizzare appearence, to the realm of the conventional stringy models. For this purpose, we employ the following *infrared universality* taking place in the regime (1.5) as far as macroscopic contours without zig-zag backtrackings[4] are concerned. The key-observation [6] is that, in this case, *the above solution of the Loop equation is supposed to be in the one infrared universality class with the unconventional (owing to eq. (2.11) below) implementation of the Nambu-Goto string* which, in turn, is supposed to be reformulated in the spirit of the 'low-energy' noncritical Polyakov's theory. Thus associated Nambu-Goto theory, presumed to possess the same UV cut off Λ, is endowed with the weight

$$w_1[\tilde{M}_\chi] = N^\chi\ exp\left(-\frac{\bar{\lambda}(\lambda)\cdot\Lambda^2}{2} A[\tilde{M}_\chi] - m_0(\lambda)L[\tilde{M}_\chi] \right)\ , \tag{2.9}$$

where $L[\tilde{M}]$ is the length of the boundary $\partial\tilde{M}$ of $\tilde{M}$, while $\bar{\lambda}(\lambda)$ and $m_0(\lambda)$ are certain functions of λ which depend on the choice of *both* the flux-tube's transverse profile *and* the prescription for the regularization of the string fluctuations. (More refined analysis [6] demonstrates that the simple $L[\tilde{M}_\chi]$-dependence of the subleading boundary-contribution is valid for $< W_C >$ only provided that all the loop's *self*intersections, if present, are *point-like* from the low-energy viewpoint.) Let us also remark that the validity of the considered infrared equivalence implies that the contour C is *macroscopic* in the concrete sense of the following twofold constraint. Firstly, except for the $1/\Lambda$-vicinity of the nontrivial (point-like) selfintersections of C, the distance $|\mathbf{x}(s)-\mathbf{x}(s')|$ is much larger than the flux-tube's width $\sqrt{<\mathbf{r^2}>} \sim \Lambda^{-1}$, provided that the length of the corresponding segment of the boundary $C = \partial\tilde{M}$ is much larger than Λ^{-1}:

$$\int_{s'}^{s} dt\ \sqrt{\dot{x}^2_\mu(t)}\ >> \Lambda^{-1} \qquad \Longrightarrow \qquad |\mathbf{x}(s)-\mathbf{x}(s')| >> \Lambda^{-1}\ . \tag{2.10}$$

Secondly, we always presume that, once $C_{xx} = C_{xy}C_{yx}$ (where $\mathbf{x}(s) = \mathbf{y}(s'),\ s \neq s'$) satisfies the above condition, then both C_{xy} and C_{yx} comply with this condition as well. It is also noteworthy that the pattern (2.9) allows to make a direct contact [6] with the proposal of [5].

[3]In fact, the pattern (2.8) is reminiscent of (but *not* equivalent to) the *ad hoc* smearing [7] of the $m_0 = 0$ option of the Nambu-Goto weight (2.9). Complementary, the weight (2.8) can be viewed as a specific implementation of the general confining string Ansatz [9] which, in turn, is routed in the abelian Kalb-Ramond pattern [12].

[4] The prescription, to implement the mandatory backtracking invariance of the YM_D loop-averages $< W_C >$, is given in [6].

2.2 The consistency with the full Dyson-Schwinger chain.

Let us now turn to the restrictions which are necessary to impose, for the (multiloop generalization of the) solution (2.3)/(2.6) to be consistent with large N Loop equation (1.3) (and, more generally, with the full chain of the $U(N)$ loop equations). We refer the reder to [6] for the details and here simply present the outcome.

To begin with, one observes that in the $N=1$ case the reduced eq. (2.1) becomes the *exact* $U(1)$ Loop equation valid for an *arbitrary* selfintersecting contour C. Furthermore, one can easily verify that the $N=1$ Ansatz (2.3)/(2.6) is the exact solution of the entire abelian Dyson-Schwinger chain. The subtle point is that the considered stringy system describes, for $g^2 > g^2_{cr}$, the electrodynamics enriched with the monopoles (similarly to the abelian systems discussed in [9]) in the SC phase where the latter are supposed to condense. Another nontrivial point is that the choice of a particular zero-mode solution $w_2^{(0)}[\tilde{M}(C)]$ is tantamount to the particular choice of the monopole's action.

Next, consider the case of sufficiently large N when the Loop equation (1.3) is the adequate leading approximation. Then, there are three major constraints to ensure that there is such a judicious regularization of the full eq. (1.3) (and of the entire chain) which makes the latter equation consistent with the Υ_0-solution (2.3)/(2.6). First of all, one has to impose the conditions (1.5) which maintain that the characteristic amplitude of the string's fluctuations (estimated by the same token as in [11]) is much larger than the vortex width $\sqrt{<\mathbf{r^2}>} \sim \Lambda^{-1}$. In this way, we render unobservable the quasi-contact interactions (additional to the flux-tube's selfenergy) between the elementary YM vortices. In turn, the necessity of this suppression is foreshadowed by the abelian nature of the latter interactions as it is clear from the above discussion of the $N=1$ case. Secondly, in the regime (1.5), the consistency with the Dyson-Schwinger chain unambiguously fixes the N-dependence of the $U(N)$ solution in the form of the 't Hooft factor N^{2-2h-b} (remaining undetermined from eq. (2.1) alone). Actually, for the number of the boundary loops $b \geq 2$ and large $N \neq \infty$, one is to impose the stricter condition to suppress as $e^{-\beta N}$ (with some $\beta > 0$) all kinds of the quasi-contact interactions. Both the characteristic size (understood in the sense of eq. (2.10)) of each loop, presumed to be devoid of $1d$ selfintersections, and the minimal distance, between any two different contours, should be of order of N^{α} with some $\alpha > 0$.

The third condition implies that, despite its familiar appearence, the system (2.3)/(2.9) is *not* entirely conventional at least for sufficiently large $N \geq 2$. Within the $1/N$ SC expansion, for the consistent regularization of the Loop equation (1.3) to exist, the confining solution should mandatory result in *the physical string tension σ_{ph} which is of order of (or, when $\lambda \to \infty$, much larger than) the UV cut off Λ squared*,

$$\Lambda^2 = O(\sigma_{ph}) \qquad ; \qquad \sigma_{ph}^{(sc)} = \left(\frac{\bar{\lambda}(\lambda)}{2} - \zeta_D\right)\Lambda^2 \ . \tag{2.11}$$

Similarly to [11], in the $N \to \infty$ semiclassical approximation $\sigma_{ph}^{(sc)}$ for σ_{ph}, the $D \geq 3$ entropy contribution $\delta\sigma_{ent} = -\zeta_D \Lambda^2$ (due to the regularized transverse string fluctuations) is represented by the λ-*independent* constant ξ_D computed in [6]. In particular, eq. (2.11) implies that, motivating the constraint (1.2), the coupling $\bar{\lambda}(\lambda)$ in eq. (2.9) is constrained to be sufficiently large.

At first glance, the Λ^2-scaling[5] (2.11) looks like a rather unnatural condition. On the second thought, one could expect this constraint beforehand: the strongly coupled $D=4$ YM_4 system (1.1) can be reinterpreted as a local *prototype* [5] of the effective low-energy theory for the gauge system (1.1) in the $\lambda \to 0$ regime with the asymptotic freedom in the UV domain. In this

[5]It is this scaling that hinders the direct application of the considered construction to the $\lambda \to 0$ regime.

perspective, in eq. (2.11) the cut off Λ is to be identified with the confinement-scale which (in the $D=4$ system (1.1) with $\lambda \rightarrow 0$ and the new UV cut off $\bar{\Lambda} >> \Lambda$) is supposed to be of order of the lowest glueball mass. From the results obtained, the considered confinement-scale can be complementary viewed as the scale where the logarithmic renormgroup flow (of the running coupling constant) freezes.

Furthermore, according to the arguments of [6], the Λ^2-scaling (2.11) is sufficient for the system (2.3)/(2.9) to suppress the outgrowth of the microscopic *baby-universes* so that the Gauge String avoids the branched polymer phase. This matches the observation that the full-fledged quantum analysis of the stringy system (2.3)/(2.9) requires to reformulate the latter as a somewhat unconventional string theory as it suggested by the concept of the Pauli-Villars regularization. Possessing the UV cut off $\tilde{\Lambda} >> \sqrt{\sigma_{ph}} \sim \Lambda$, this theory should not exhibit propagating degrees of freedom at the 'short distance' scales $<< 1/\sqrt{\sigma_{ph}}$, while approaching the proposed implementation (2.11) of the Nambu-Goto pattern (2.3)/(2.9) at the scales larger than $1/\sqrt{\sigma_{ph}}$.

3 The regime (1.6) of the quasi-static YM vortex.

The land-mark of the so far considered regime (1.5) is that the quasi-contact interactions between the elementary YM vortices, being discarded by the sheer Nambu-Goto pattern (2.9), are unobservable. In fact, it is tantamount [5, 6] to the unobservability of the choice of the particular action and nonabelian group defining a given dual gauge theory. The latter choice can be traced back most transparently in the extreme SC limit (1.6) where the $YM_D \rightarrow YM_2$ dimensional reduction (see eq. (3.3) below), anticipated in [5], takes place. To reveal this phenomenon in the simplest setting, one is to take advantage of the fact that the $N=1$ option of the Ansatz (2.3)/(2.8) correctly reproduces the SC expansion in the $D \geq 2$ $U(1)$ gauge theory (1.1) modified by the presence of the monopoles in $D \geq 3$. In particular, the latter $N=1$ Ansatz is applicable not only to the regime (1.5) but also to the extreme SC limit (1.6). As a result, the abelian analysis provides with the following general motivations presumably valid for $\forall N \geq 1$.

We proceed by noticing that, in the limit (1.6), the leading asymptotics of the average $< W_C >$ is supposed to be given by the contribution of the saddle-point YM vortex corresponding to the minumal area worldsheet(s) $\tilde{M}_{min}(C)$ with the characteristic radius of curvature

$$\mathcal{R}(\gamma)\Lambda \longrightarrow \infty , \tag{3.1}$$

i.e. being (at any point $\gamma \equiv (\gamma_1, \gamma_2)$ of the surface) much larger than the flux-tube's width $\sim \Lambda^{-1}$. Let the saddle-point (minimal area) worldsheet(s) $\tilde{M}_{min}(C)$ possess in $\mathbf{R^D}$ the support $T_{min} = T_{min}(C)$ which, for simplicity, is presumed to be *unique* for any given C in question. Also, to avoid certain technical complications, we will presume that $T_{min}(C)$ can be embedded into a $2d$ manifold $\mathbf{B^2}$ diffeomorphic to $\mathbf{R^2}$. Then, the SC asymptotics (1.6) of the $N=1$ Ansatz (2.3)/(2.8) can be written as the $N=1$ option of the general pattern

$$< W_C > \Big|_{YM_2(T_{min})} = \sum_{\{R_q\}} F(\{R_q\})\ exp\left(-\frac{\xi\lambda\Lambda^2}{2} \sum_q\ C_2(R_q)\ \bar{A}_q\right) \tag{3.2}$$

where the parameter ξ is defined by eq. (3.5) below, $C_2(R)$ is the eigenvalue of the second $U(N)$ Casimir operator associated to the representations R (so that $C_2(n) = n^2$ in the $U(1)$ case), while $\bar{A}_q$ is the area $A[T_q]$ of the qth window T_q of the support $T_{min}(C)$. As for the origin of T_q, the support $T_{min}(C)$ is devided into the union $\{T_q;\ \sum_q \bar{A}_q = A[T_{min}(C)]\}$ of the disjoint windows T_q after cutting along the loop C. Remark that each domain $T_q(\{C_k\})$ is assigned with

some representation R_q (entering $C_2(R_q)$) of the Lie group in question, and the sum in eq. (3.2) runs over all admissible $\{R_q\}$-assignements satisfying the (nonabelian) *fusion-rule* algebra. The latter is imposed by the $\{\bar{A}_q\}$-independent function $F(\{R_q\})$.

Upon a reflection, eq. (3.2) yields the average $< W_C > |_{YM_2(T_{min})}$ evaluated in the continuous $D = 2$ YM_2 theory (1.1) conventionally defined on $T_{min}(C)$ as on the base-space. Altogether, it motivates the following statement (substantiated in [5, 6] in the alternative way). Keeping the conditions (1.6) fulfilled, let both the coupling constant λ and $(\mathcal{R}(s)\Lambda)$ be much larger than N^2. Given the above conditions and summing[6] up the leading (with respect to the $1/\lambda$- and $1/(<\mathcal{R}(\gamma)> \Lambda)-$expansions) subseries in $1/N^2$, the pattern of the $D \geq 3$ YM_D averages $< W_C > |_{YM_D}$ is supposed to exhibit the reduction

$$< W_C > |_{YM_D} \quad \longrightarrow \quad < W_C > |_{YM_2(T_{min})} \qquad if \qquad \lambda, \; (\mathcal{R}(s)\Lambda) \quad >> N^2 \; , \tag{3.3}$$

where the YM_2 coupling constant g_{YM_2} is related to the original $D \geq 3$ YM_D constant g_{YM_D} via the rescaling

$$g^2_{YM_2} = \xi \; g^2_{YM_D} \; \Lambda^{D-2} = \xi \; \frac{\lambda}{N} \; \Lambda^2 \tag{3.4}$$

that can be interpreted as the $YM_D \to YM_2$ *dimensional reduction* implemented modulo the auxiliary ξ-factor. In turn, the parameter[7] ξ, reflecting the ambiguity of the UV regularization, is related to the smearing function (2.2),

$$\xi = \int d^2 z \; \mathcal{G}(\mathbf{z}^2) \sim 1 \qquad ; \qquad \sigma_0 = \frac{\xi \lambda \Lambda^2}{2} \; . \tag{3.5}$$

that defines the corresponding bare string tension σ_0. Below, we will prove the large N variant of the SC asymptotics (3.3) directly from the Loop equation (1.3), while now it is appropriate to discuss some implications of eq. (3.3).

To begin with, the YM_2-average in the r.h. side of eq. (3.3) can be reformulated in the manifestly stringy terms employing the construction of [3, 5]. To draw a particularly transparent consequence of the SC asymptotics (3.3), consider the average $< W^R_{C'} >$ of the Wilson loop in any *(anti)chiral* representation $R \in Y^{(N)}_n$ of the $U(N)$ group. Then, eq. (3.3) implies that the leading asymptotics of the physical string tension (associated to $< W^R_{C'} >$) is proportional,

$$\sigma_{ph}(R) \quad \longrightarrow \quad \frac{C_2(R)}{N} \, \sigma_0 \qquad ; \qquad R \in Y^{(N)}_n \; , \tag{3.6}$$

to the eigenvalue $C_2(R)$ of the second $U(N)$ Casimir operator. Then, to compare this pattern with the classification of the (dual) abelian superconductors, we observe that $C_2(R) \geq |n(R)|$, where $n(R) = \sum_{i=1}^N n_i$ measures the total amount of the elementary $U(N)$ fluxes. (Conventionally, N nondecreasing integers n_i parametrize a generic $U(N)$ representation R so that $C_2(R) \sim N$ once $|n(R)| \sim N^0$.) As a result, the averaged force between the collinear elementary YM vortices is *repulsion*. The Gauge String, associated to the gauge theory (1.1), corresponds therefore to the type-II superconductor.

To make contact with the $N \geq 2$ Ansatz (2.3)/(2.8), remark first that one obtains $\tilde{\sigma}_{ph}(R) \to |n(R)|\sigma_0$, when $\lambda \to \infty$ in the regime (1.5). Complementary, the SC asymptotics (1.6) of the $N \geq 2$ system (2.3)/(2.8) complies with eq. (3.3) once the saddle-point flux-tube is essentially nonselfoverlapping. In turn, it ensures that the quasi-contact interactions (between the elementary YM vortices) are unobservable in this case which, in the leading order, leaves the sheer $m_0 = 0$, $\chi = 1$ Nambu-Goto pattern (2.9) applied to the minimal area surface $\tilde{M}_{min}(C)$.

[6] When both λ and $(\mathcal{R}(s)\Lambda)$ are large but $\leq N^2$, in the r.h. side of eq. (3.3) one is to retain only the leading $O(N)$ order of the $1/N$ expansion of the $YM_2(T_{min})$ average.

[7] Furthermore, one can argue that (at least when $\lambda \sim 1$) the factor ξ, reflecting the $D \geq 3$ regularization ambiguity, is *of order of unity* once the solution (2.8) describes stable, rather than metastable, stringy excitations.

3.1 Sketching the derivation of the SC asymptotics (3.3).

For simplicity, we restrict our discussion to the $N \to \infty$ case imposing, as previously, that the support $T_{min}(C)$ of the 'minimal area' flux-tube (spanned by C) can be embedded into a curved two-dimensional space $\mathbf{B^2}$ diffeomorphic to the $2d$ plane $\mathbf{R^2}$. In this case, the generally covariant extension of the $2d$ YM_2 theory (1.1) can be introduced on $\mathbf{B^2}$ in the standard way so that the pattern of the r.h. side of eq. (3.3) can be borrowed from the $\mathbf{B^2} = \mathbf{R^2}$ computations [10]. Let a judicious smearing $< W_C > |_{YM_2(T_{min})} \longrightarrow < W_C > |^{(reg)}_{YM_2(T_{min})}$,

$$< W_C > |^{(reg)}_{YM_2(T_{min})} \Big/ < W_C > |_{YM_2(T_{min})} \;=\; 1 + O\Big((< \mathcal{R}(s) > \Lambda)^{-\alpha}\Big) \quad , \quad \alpha > 0 \, , \tag{3.7}$$

is unobservable in the regime (1.6). The statement is that thus regularized r.h. side of eq. (3.3) satisfies the (regularized) D-dimensional Loop equation (1.3). The key-point of the proof is that, when the considered pattern is plunged (in the regime (1.6)) into eq. (1.3), the latter equation is reduced to the *two*-dimensional Loop equation conventionally associated to the $D = 2$ YM_2 theory (1.1)/(3.4) on $\mathbf{B^2}$. In turn, the resulting $D = 2$ Loop equation is fulfilled by the very construction (3.7) of the smearing for the SC asymptotics (3.3).

To more explicit, in eq. (3.2), the suitable smearing prescription is to substitute each particular $2A_q$ by the $\tilde{M} \to T_q$ option of the bilocal integral (2.8). Being understood in the stronger sense of eq. (3.3) (complemented by the constraint $\bar{A}_q \Lambda^2 >> N^4$, for $\forall q$, together with the two more requirements formulated below), the condition (3.1) justifies the formal substitution

$$\Lambda^2 \mathcal{G}(\Lambda^2(\mathbf{x}-\mathbf{y})^2) \longrightarrow \xi \; \delta_2^w(\mathbf{x}(\gamma) - \mathbf{y}(\gamma')) \, , \tag{3.8}$$

where $\delta_2^w(\mathbf{x}-\mathbf{y})$ is the 2-dimensional delta-function on $T_{min}(C)$, and ξ is defined in eq. (3.5). One is to require also that (at any point of $T_{min}(C)$) the line in the normal direction either does not have the second intersection with $T_{min}(C)$ or the second intersection takes place at a distance $>> \Lambda^{-1}$. Additionally, for the validity of eq. (3.7), we have to ensure the stricter suppression of the next-to-leading boundary contribution, proportional to the effective perimeter-mass $m_0(\lambda)$ (which dependence on the smearing function (2.2) is evaluated in [6]), that introduces an extra constraint on $\mathcal{G}(\mathbf{z}^2)$. Altogether, the imposed conditions allow to substantiate eq. (3.7).

Next, to justify the asserted dimensional reduction of the Loop equation (1.3), one is to synthesize the following two observations. Firstly, the above smearing prescription together with the condition (3.1) ensure that, acting on the smeared pattern (3.2), the D-dimensional Loop operator $\hat{\mathcal{L}}^\nu(\mathbf{x}(s))$ reduces to the *two*-dimensional one

$$\hat{\mathcal{L}}^\alpha(\mathbf{x}(s)) = \frac{1}{\sqrt{\hat{g}(\mathbf{x})}} \; \partial_\beta^{\mathbf{x}(s)} \; \sqrt{\hat{g}(\mathbf{x})} \; \hat{g}^{\beta\gamma}(\mathbf{x}) \; \hat{g}^{\alpha\lambda}(\mathbf{x}) \; \frac{\delta}{\delta\sigma^{\gamma\lambda}(\mathbf{x}(s))} \tag{3.9}$$

where $\hat{g}_{\alpha\beta}(\mathbf{x})$ is the $2d$ metric on $\mathbf{B^2}$, and $\hat{g}(\mathbf{x}) \equiv det_{\{\alpha\beta\}}[\hat{g}_{\alpha\beta}(\mathbf{x})]$. The asserted reduction[8] takes place due to the evident property of the 2-tensor $\mathcal{K}_{\mu\nu}(\mathbf{x}(s))$, resulting after the action of the 'D-dimensional' Mandelstam derivative $\delta/\delta\sigma^{\mu\nu}(\mathbf{x}(s))$ onto the smeared exponent of eq. (3.2). The constraint (3.1) implies that the tensor $\mathcal{K}_{\mu\nu}(\mathbf{x}(s))$ has nonvanishing components only in the subspace spanned by the two Zweibein D-vectors $\mathbf{e}^\beta(\mathbf{x}) \in \mathbf{t_x B^2}$ beloning to the tangent space $\mathbf{t_x B^2}$ of $\mathbf{B^2}$ at a given point $\mathbf{x} = \mathbf{x}(s)$ so that $(\mathbf{e}^\alpha(\mathbf{x})|\mathbf{e}^\beta(\mathbf{x})) \equiv \sum_\mu e^\alpha_\mu(\mathbf{x}) e^\beta_\mu(\mathbf{x}) = \hat{g}^{\alpha\beta}(\mathbf{x})$.

[8]Similar reduction, but in the context of the $\lambda \to 0$ YM_D theory (2.8) considered *prior* to the UV regularization, is discussed in [16].

The second observation is that, being devided by Λ^{D-2}, the regularization (2.2) of the D-dimensional δ_D-function (in the r.h. side of the properly regularized eq. (1.3)) simultaneously can be reinterpreted as the smearing (3.8) of the 2-dimensional $\xi\delta_2(..)$-function on $\mathbf{B^2}$, where ξ is defined by eq. (3.5). (Complementary, the value of $\xi\tilde{g}^2\Lambda^{D-2}$ is equal to the coupling constant (3.4) in the YM_2 theory entering the asymptotics (3.3).) In turn, the regularized $\delta_2(..)$-function refers to the r.h. side of the Loop equation conventionally associated to the generally covariant extension of the $2d$ YM_2 theory (1.1)/(3.4) defined on the $2d$ manifold $\mathbf{B^2}$ into which the support $T_{min}(C)$ is embedded. As for the economic regularization of the r.h. side of eq. (1.3), the following simple prescription is sufficient in the regime (1.6) provided the relevant contours are presumed to be *macroscopic* (according to the definition given prior to eq. (2.10)). In this case, one can introduce the regularization *separately* for the $1/\Lambda$-vicinities of the nontrivial (point-like) selfintersections of C (i.e. when $s' \neq s$) and for the remaining trivial selfintersections (i.e. when $s' = s$). Namely, consider for simplicity a macroscopic contour C with a single nontrivial selfintersection at $\mathbf{x}(s_1) = \mathbf{y}(s_2)$. For $\mathbf{x}(s) = \mathbf{x}(s_1)$, the r.h. side of eq. (1.3) is given by the weighted sum $a_1 < W_{C_{xx}} >_\infty + a_2 < W_{C_{xy}} >_\infty < W_{C_{yx}} >_\infty$ where the weights a_1, a_2 can be represented by the judiciously selected portions [10] of the contour integral along C. Then, the regularization prescription is to perform the substitution (2.2) separately within a_1 and a_2.

Summarizing, after the proper smearing (3.7), the Ansatz (3.3) indeed reduces thus regularized D-dimensional eq. (1.3) to the regularized two-dimensional Loop equation defined on $\mathbf{B^2}$. To finally put all the pieces together, one can show that, for the macroscopic loops in the considered regime (1.6), the regularization of the latter $2d$ equation is *unobservable* that happily matches the complementary unobservability (3.7). This matching implies that thus regularized $2d$ Loop equation merges[9] with the equation [10] derived for the YM_2 theory (1.1)/(3.4) with the *infinite* (in the units of $(g^2_{YM_2}N)^{1/2}$) UV cut off $\tilde{\Lambda} \rightarrow \infty$ which, in turn, justifies the properly smeared asymptotics (3.3) as the solution of eq. (1.3).

4 Conclusions.

Hopefully, the present analysis can be included, as a building block, to a larger scheme which would make accessible the solution of the $D = 4$ Loop equation (1.3) in the most interesting $\lambda \rightarrow 0$ phase of the YM_4 theory (1.1) (with the asymptotic freedom in the UV domain). The promising sign is that the derived $YM_D \rightarrow YM_2$ dimensional reduction (3.3) is reminiscent of the one in the model [17] which is rather popular for the approximation of the lattice results presumably associated to the gauge theory (1.1). Complementary, consider the low-energy dynamics of the $U(\infty)$ or $SU(\infty)$ YM_4 theory (1.1) (with $\lambda \rightarrow 0$ and the new UV cut off $\bar{\Lambda} >> \Lambda$) at the confinement-scale where the logarithmic renormgroup flow, valid in the UV domain, is presumably freezed. Then, the latter infrared dynamics is supposed to be correctly described by the $N \rightarrow \infty$ limit of the proposed implementation (2.11) of the Nambu-Goto system (2.3)/(2.9), provided we are dealing with such correlators where the quasi-contact interactions of the fat YM vortices are unobservable. This statement is supported by the observation [6] that the large N limit of the considered Nambu-Goto sum is expected to be *common* for the infrared description of *any* $D \geq 3$ $U(\infty)$ or $SU(\infty)$ pure gauge system with an arbitrary polynomial (in terms of $F_{\mu\nu}$) lagrangian [5] providing with the N^2-scaling of the free energy.

[9]The only subtlety is due to certain extra 'anomaly term' arising when the original variant of area-derivative (with the area-variation $|\delta\sigma_{\mu\nu}(\mathbf{x})| << \Lambda^{-1}$) is to be transformed into the one with $|\delta\sigma_{\mu\nu}(\mathbf{x})| >> \tilde{\Lambda}^{-1}$. In our computation, this perimeter-dependent anomaly can be independently recovered comparing the action of the area-derivative on the bilocal pattern (2.8) in both of the relevant limiting cases.

Finally, it is noteworthy that the regime (1.6) is reminiscent of the one which independently appeared within the alternative stringy description [14] (see also [15]) of the pure gauge theories which is based on the conjecture [13] about the so-called *AdS/CFT correspondence*. Given any macroscopic nonselfintersecting contour, in the extreme $D=4$ SC regime $\lambda >> 1$, the purported $\lambda\Lambda^2$-scaling [14] of the physical string tension is consistent with the prediction [5] of our formalism. For an arbitrarily selfintersecting loop, the predictions of [14] are still missing. The pattern of the SC asymptotics (3.3), being actually valid for a generic YM_D system with the lagrangian polynomial in term of $F_{\mu\nu}$, imposes the concrete test (to be compared with the arguments of [15]) on the applicability of [14].

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